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3D-xy critical properties of $YBa_2Cu_4O_8$ and magnetic field induced 3D to 1D crossover

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Abstract. - We present and analyze reversible magnetization data of a $YBa_2Cu_4O_8$ single crystal to explore the evidence for 3D-xy critical behavior and a magnetic field induced 3D-1D crossover. Remarkable consistency with these phenomena is observed in agreement with a magnetic field induced finite size effect whereupon the correlation length transverse to the applied magnetic field cannot grow beyond the limiting magnetic length scale $L_H = (\Phi_0/(aH))^{1/2}$. By applying the appropriate scaling form we obtain the zero field critical temperature, the 3D to 1D crossover and the vortex melting line and the universal ratios of the related scaling variables. Accordingly there is no continuous phase transition in the (H_c, T)-plane along the H_{c2}-lines as predicted by the mean-field treatment.

In this study we present and analyze reversible magnetization data of a $YBa_2Cu_4O_8$ single crystal to explore the evidence for 3D-xy critical behavior and a magnetic field induced 3D-1D crossover. We observe remarkable consistency with these phenomena. Since near the zero field transition temperature T_c thermal fluctuations are expected to dominate [1–5], Gaussian fluctuations point to a magnetic field induced 3D to 1D crossover [6], whereby the effect of fluctuations is enhanced, it appears inevitable to take thermal fluctuations into account. Indeed, invoking the scaling theory of critical phenomena we show that the data are inconsistent with the traditional mean-field interpretation. On the contrary, we observe agreement with a magnetic field induced finite size effect, whereupon the correlation length transverse to the magnetic field H_i, applied along the i-axis, cannot grow beyond the limiting magnetic length

$$L_{H_i} = (\Phi_0/(aH_i))^{1/2}, \qquad (1)$$

with $a \simeq 3.12$ [7]. L_{H_i} is related to the average distance between vortex lines. Indeed, as the magnetic field increases, the density of vortex lines becomes greater, but this cannot continue indefinitely, the limit is roughly set on the proximity of vortex lines by the overlapping of their cores. This finite size effect implies that in type II superconductors, superconductivity in a magnetic field is confined to cylinders with diameter L_{H_i} [5, 8]. Accordingly, below T_c there is the 3D to 1D crossover line

$$H_{pi}(T) = \left(\Phi_0/\left(a\xi_{j0}^{-}\xi_{k0}^{-}\right)\right)(1 - T/T_c)^{4/3}, \qquad (2)$$

with $i \neq j \neq k$. $\xi_{i0,j0,k0}^{-}$ denotes the critical amplitudes of the correlation lengths below $(-)$ T_c along the respective axis. It circumvents the occurrence of the continuous phase transition in the (H_c, T)-plane along the H_{c2}-lines predicted by the mean-field treatment [11]. Indeed, the relevance of thermal fluctuations emerges already from the reversible magnetization data shown in Fig. 1. As a matter of fact, the typical mean-field behavior [11], whereby the magnetization scales below T_c linearly with the magnetic field, does not emerge.

The $YBa_2Cu_4O_8$ single crystal investigated in this work was fabricated by a high-pressure synthesis method described in detail elsewhere [9, 10]. The volume of the nearly rectangular shaped sample is estimated to $3.9 \cdot 10^{-4}$ cm^3. The magnetic moment was measured by a commercial Quantum Design DC-SQUID magnetometer MPMS XL with installed RSO option allowing to achieve a resolution of 10^{-8} emu. For the experiment the applied magnetic field was oriented along the c-axis of the crystal. For different magnitudes of the field temperature dependent magnetization curves where measured. The zero-field

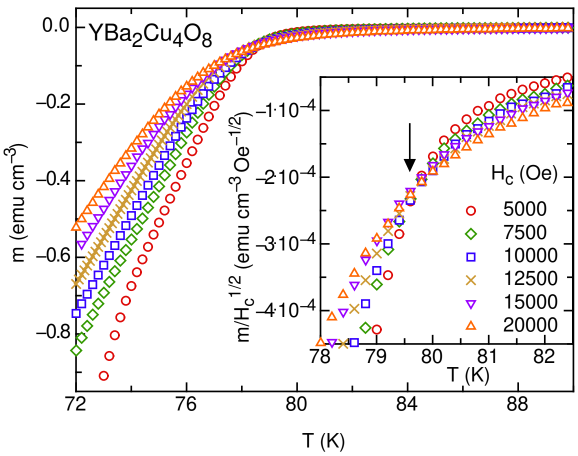

Fig. 1: Reversible magnetization m *vs.* T of a $YBa_2Cu_4O_8$ single crystal for magnetic fields H_c applied along the c-axis. The inset shows $m/H_c^{1/2}$ *vs.* T. The arrow indicates the crossing point which yields the estimate $T_c \simeq 79.6K$.

cooled (ZFC) and field cooled (FC) data have been compared in order to probe the reversible magnetization only. The superconducting susceptibility was finally obtained by correcting the measured data for the normal state and sample holder contributions. In Fig. 1 we depicted some of the measured magnetization curves m *vs.* T for magnetic fields H_c applied along the c-axis. On a first glance the data falls on rather smooth curves, revealing that the extraction of critical and crossover behavior requires a rather detailed analysis. When thermal fluctuations dominate and the coupling to the charge is negligible the magnetization per unit volume, $m = M/V$, adopts the scaling form [1–4]

$$\begin{aligned} \frac{m}{TH^{1/2}} &= -\frac{Q^{\pm}k_B\xi_{ab}}{\Phi_0^{3/2}\xi_c}F^{\pm}(z),\ F^{\pm}(z) = z^{-1/2}\frac{dG^{\pm}}{dz}, \\ z &= x^{-1/2\nu} = \frac{\left(\xi_{ab0}^{\pm}\right)^2|t|^{-2\nu}H_c}{\Phi_0}. \end{aligned} \tag{3}$$

$Q^{\pm}$ is a universal constant and $G^{\pm}(z)$ a universal scaling function of its argument, with $G^{\pm}(z=0)=1$. $\gamma = \xi_{ab}/\xi_c$ denotes the anisotropy, ξ_{ab} the zero-field in-plane correlation length and H_c the magnetic field applied along the c-axis. In terms of the variable x the scaling form (1) is similar to Prange's [12] result for Gaussian fluctuations. Approaching T_c the in-plane correlation length diverges as

$$\xi_{ab} = \xi_{ab0}^{\pm}|t|^{-\nu},t = T/T_c - 1,\ \pm = sgn(t). \tag{4}$$

Supposing that 3D-xy fluctuations dominate the critical exponents are given by [13]

$$\nu \simeq 0.671 \simeq 2/3,\ \alpha = 2\nu - 3 \simeq -0.013, \tag{5}$$

and there are the universal critical amplitude relations [1–5,13]

$$\frac{\xi_{ab0}^{-}}{\xi_{ab0}^{+}} = \frac{\xi_{c0}^{-}}{\xi_{c0}^{+}} \simeq 2.21,\ \frac{Q^{-}}{Q^{+}} \simeq 11.5,\ \frac{A^{+}}{A^{-}} = 1.07, \tag{6}$$

and

$$\begin{aligned} A^{-}\xi_{a0}^{-}\xi_{b0}^{-}\xi_{c0}^{-} &\simeq A^{-}\left(\xi_{ab0}^{-}\right)^2\xi_{c0}^{-} = \frac{A^{-}\left(\xi_{ab0}^{-}\right)^3}{\gamma} \\ &= \left(R^{-}\right)^3, R^{-} \simeq 0.815. \end{aligned} \tag{7}$$

$A^{\pm}$ is the critical amplitude of the specific heat singularity, defined as

$$c = \frac{C}{Vk_B} = \frac{A^{\pm}}{\alpha}|t|^{-\alpha} + B, \tag{8}$$

where B denotes the background. Furthermore, in the 3D-xy universality class T_c, ξ_{c0}^{-} and the critical amplitude of the in-plane magnetic field penetration depth λ_{ab0} are not independent but related by the universal relation [1–4,13],

$$k_BT_c = \frac{\Phi_0^2}{16\pi^3}\frac{\xi_{c0}^{-}}{\lambda_{ab0}^2} = \frac{\Phi_0^2}{16\pi^3}\frac{\xi_{ab0}^{-}}{\gamma\lambda_{ab0}^2}. \tag{9}$$

Furthermore, the existence of the magnetization at T_c, of the penetration depth below T_c and of the magnetic susceptibility above T_c imply the following asymptotic forms of the scaling function [1–5]

$$\begin{aligned} Q^{\pm}\frac{1}{\sqrt{z}}\frac{dG^{\pm}}{dz}\bigg|_{z\to\infty} &= Q^{\pm}c_{\infty}^{\pm}, \\ Q^{-}\frac{dG^{-}}{dz}\bigg|_{z\to 0} &= Q^{-}c_0^{-}\left(\ln z + c_1\right), \\ Q^{+}\frac{1}{z}\frac{dG^{+}}{dz}\bigg|_{z\to 0} &= Q^{+}c_0^{+}, \end{aligned} \tag{10}$$

with the universal coefficients

$$\begin{gathered} Q^{-}c_0^{-} \simeq -0.7,\ Q^{+}c_0^{+} \simeq 0.9,\ Q^{\pm}c_{\infty}^{\pm} \simeq 0.5, \\ c_1 \simeq 1.76. \end{gathered} \tag{11}$$

We are now prepared to analyze the magnetization data. To estimate T_c we note that according to Eqs. (3), (10) and (11) the plot $m/H_c^{1/2}$ *vs.* T should exhibit a crossing point at T_c because $m/TH_c^{1/2}$ tends to the value $m/T_cH_c^{1/2} = -0.5k_B\gamma\Phi_0^{-3/2}$. The insert in Fig. 1 reveals that there is a crossing point near $T_c \simeq 79.6$ K. Given this estimate consistency with 3D-xy critical behavior then requires according to the scaling form (3) that the data plotted as $m/(TH^{1/2})$ *vs.* $tH^{-1/2\nu} \simeq tH^{-3/4}$ should collapse near $tH^{-3/4} \to 0$ on a single curve. Evidence for this collapse emerges from Fig. 2 with $T_c \simeq 79.6$ K. Considering the limit $z \to 0$ below T_c the appropriate scaling form is

$$\frac{m}{T} = -\frac{Q^{-}c_0^{-}k_B}{\Phi_0\xi_c^{-}}\left(\ln\left(\frac{H\left(\xi_{ab}\right)^2}{\Phi_0}\right) + c_1\right), \tag{12}$$

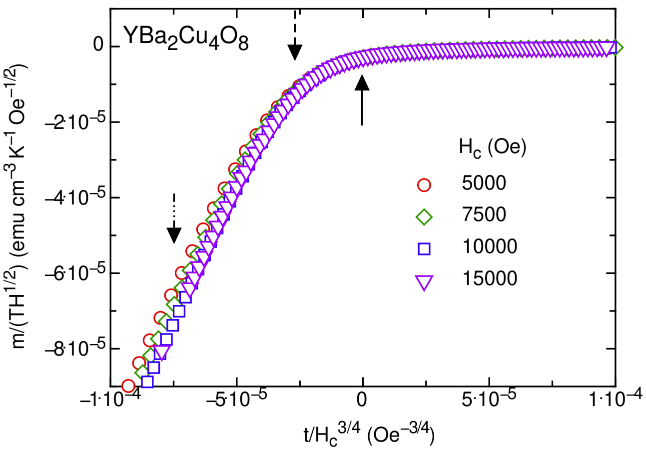

Fig. 2: $m/\left(TH_c^{1/2}\right)$ *vs.* $t/H^{3/4}$ for a $YBa_2Cu_4O_8$ single crystal with $T_c = 79.6$ K. The full arrow marks the zero field critical temperature T_c, the dashed arrow the 3D to 1D crossover and the dotted arrow the vortex melting line.

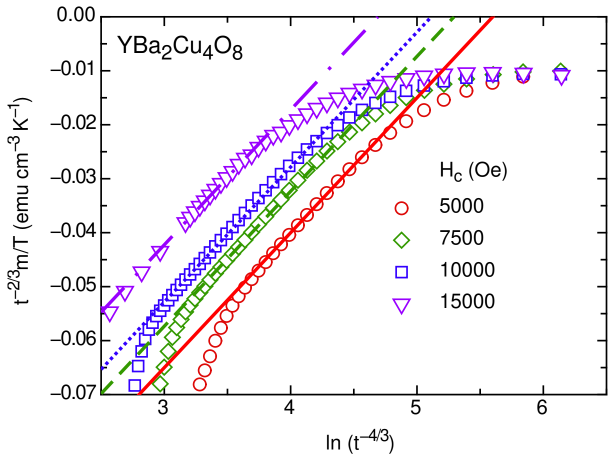

Fig. 3: $|t|^{-2/3}\, m/T$ *vs.* $\ln\left(|t|^{-4/3}\right)$ for a $YBa_2Cu_4O_8$ single crystal according to Eq. (13). The lines are fits to the rescaled magnetization data. Here $\xi_{ab0}^2\,|t_p|^{-4/3} = \Phi_0/aH_c$.

according to Eqs. (3), (10) and (11). Thus, given the magnetization data of a homogenous system, attaining the limit $z = H\left(\xi_{ab0}^{\pm}\right)^2 |t|^{-2\nu}/\Phi_0 << 1$, the growth of ξ_{ab} and ξ_c is unlimited and estimates for ξ_{c0}^- and ξ_{ab0}^- can be deduced from

$$|t|^{-2/3}\frac{m}{T} = -\frac{Q^- c_0^- k_B}{\Phi_0 \xi_{c0}^-}\left(\ln\left(\frac{H\left(\xi_{ab0}^-\right)^2 |t|^{-4/3}}{\Phi_0}\right) + c_1\right). \tag{13}$$

In Fig. 3 we depicted $|t|^{-2/3}\, m/T$ vs. $\ln\left(|t|^{-4/3}\right)$. From the straight lines we obtain

$$-\frac{Q^- c_0^- k_B}{\Phi_0 \xi_{c0}^-} \simeq 0.025, \tag{14}$$

and with that

$$\xi_{c0}^- \simeq 1.87\ \text{Å}. \tag{15}$$

Furthermore, from $\ln\left(H_c\left(\xi_{ab0}^-\right)^2/\Phi_0\right)$ *vs.* $ln(H_c)$ we deduce

$$\xi_{ab0}^- \simeq 15.6\ \text{Å}. \tag{16}$$

For the anisotropy we obtain then the estimate

$$\gamma = \frac{\xi_{ab0}^-}{\xi_{c0}^-} \simeq 8.34, \tag{17}$$

compared to $\gamma_{ca} \simeq 13.4$, $\gamma_{ca} \simeq 14.7$ [14] and $\gamma \simeq 12.3$ [15].

To explore the magnetic field induced 3D to 1D crossover further and to probe the vortex melting line directly we invoke Maxwell's relation

$$\left.\frac{\partial\left(C/T\right)}{\partial H_c}\right|_T = \left.\frac{\partial^2 M}{\partial T^2}\right|_{H_c}, \tag{18}$$

uncovering the vortex melting transition in terms of a singularity, while the magnetic field induced finite size effect leads to a dip. These features differ drastically from the nearly smooth behavior of the magnetization. Together with the scaling form of the specific heat (Eq. (8)), extended to the presence of a magnetic field,

$$c = \frac{A^-}{\alpha}|t|^{-\alpha} f(x),\ x = \frac{t}{H^{1/2\nu}}, \tag{19}$$

we obtain the scaling form

$$\begin{aligned} TH_c^{1+\alpha/2\nu}\frac{\partial\left(c/T\right)}{\partial H_c} &= -\frac{k_B A^-}{2\alpha\nu}x^{1-\alpha}\frac{\partial f}{\partial x} \\ &= TH_c^{1+\alpha/2\nu}\frac{\partial^2 m}{\partial T^2}. \end{aligned} \tag{20}$$

In Fig. 4 we depicted $TH_c d^2m/dT^2$ *vs.* x for various magnetic fields H_c. Apparently, the data collapses reasonably well on a single curve. There is a peak and a dip marked by the arrow and the vertical line, respectively. Their occurrence differs drastically from the mean-field behavior where $\partial^2 m/\partial T^2 = 0$. The finite depth of the dip is controlled by the magnetic field induced finite size effect. It replaces the reputed singularity at T_{c2} obtained in the Gaussian approximation [12]. Note that both, the peak and the dip are hardly visible in the magnetization shown in Fig. 2. There we marked the location of the peak and dip in terms of the dashed and solid arrow. The location of the dip determines the line

$$x_p = t_p H_c^{-3/4} \simeq -2.85\cdot 10^{-5}(\text{Oe}^{-3/4}), \tag{21}$$

in the (H_c, T)-plane where the 3D to 1D crossover occurs. Along this line, rewritten in the form $H_{pc}(T) = \Phi_0/\left(a\left(\xi_{ab0}^-\right)^2\right)(1 - T/T_c)^{4/3}$, the in-plane correlation length is limited by L_{H_c}(Eq. (1)). In addition there is a peak at

$$x_m = t_m H_c^{-3/4} \simeq -8.35\cdot 10^{-5}(\text{Oe}^{-3/4}), \tag{22}$$

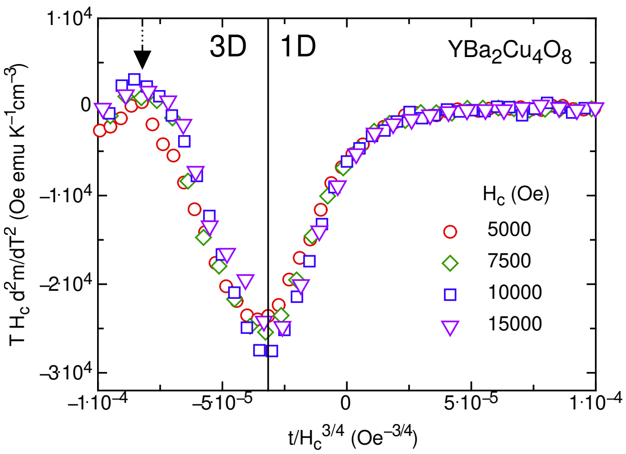

Fig. 4: TH_cd^2m/dT^2 *vs.* $x = t/H_c^{3/4}$ for a $YBa_2Cu_4O_8$ single crystal. The arrow marks the vortex melting line $x_m \simeq -8.35 \cdot 10^{-5}(\text{ Oe}^{-3/4})$ and the vertical line $x_p \simeq -2.85 \cdot 10^{-5}(\text{ Oe}^{-3/4})$ the 3D to 1D crossover line

corresponding to the vortex melting line. Rewritten in the form $H_{mc} \simeq 2.7 \cdot 10^5 (1 - T_m/T_c)^{4/3}$ (Oe) it agrees very well with the previous estimate $H_{mc} \simeq 1.8 \cdot 10^5 (1 - T_m/T_c)^{4/3}$ (Oe) of Katayama *et al.* [16] , as the temperature dependence is concerned. Accordingly, for the universal ratios of the scaling variables and the reduced temperatures at the vortex melting transition and the 3D to 1D crossover line we obtain

$$\frac{z_m}{z_p} = \left(\frac{t_p(H_c)}{t_m(H_c)}\right)^{2\nu} \simeq 0.24,$$
$$t_p(H_c)/t_m(H_c) \simeq 0.34. \tag{23}$$

This value agrees well with the estimates $t_p(H_c)/t_m(H_c) \simeq 0.3$ for a $NdBa_2Cu_3O_{7-\delta}$ single crystal [17] and $t_p(H_c)/t_m(H_c) \simeq 0.35$ for a $YBa_2Cu_3O_{6.97}$ single crystal [18] derived from the respective references. Finally, invoking the universal relation (9) we obtain with $T_c = 79.6$ K and $\xi_{c0}^- \simeq 1.87$ Å (Eq. (15)) for the critical amplitude of the in-plane magnetic field penetration depth the value $\lambda_{ab0} \simeq 1.37 \cdot 10^{-5}$cm, in reasonable agreement with the estimate $\lambda_{ab0} \simeq 1.7 \cdot 10^{-5}$cm obtained from magnetization data of polycrystalline $YBa_2Cu_4O_8$ samples [19].

We have shown that the analysis of reversible magnetization data of a $YBa_2Cu_4O_8$ single crystal provides considerable insight into the effect of thermal fluctuations and the magnetic field induced crossover. In particular we demonstrated that the fluctuation dominated regime is experimentally accessible and uncovers remarkable consistency with 3D-xy critical behavior. There is, however, the magnetic field induced finite size effect. It implies that the correlation length transverse to the magnetic field H_i, applied along the i-axis, cannot grow beyond the limiting magnetic length $L_{H_i} = (\Phi_0/(aH_i))^{1/2}$, related to the average distance between vortex lines. Invoking the scaling theory of critical phenomena clear evidence for this finite size effect has been provided. In type II superconductors it comprises the 3D to 1D crossover line $H_{pi}(T) = \left(\Phi_0/\left(a\xi_{j0}^-\xi_{k0}^-\right)\right)(1 - T/T_c)^{4/3}$ with $i \neq j \neq k$ and $\xi_{i0,j0,k0}^-$ denoting the critical amplitude of the correlation length below T_c. As a result , below T_c and above $H_{pi}(T)$ superconductivity is confined to cylinders with diameter L_{H_i}(1D). Accordingly, there is no continuous phase transition in the (H_c, T)-plane along the H_{c2}-lines as predicted by the mean-field treatment. In addition we confirmed the universal relationship between the 3D-1D crossover and vortex melting line. The universal relation (9) and Maxwell's relation (18) also imply that the effects of isotope exchange and pressure on T_c, in-plane magnetic field penetration depth, correlation lengths, specific heat, and magnetization are not independent.

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REFERENCES

[1] Hofer J., Schneider T., Singer J. M., Willemin M., and Keller H., *Phys. Rev. B*, **60** (1999) 1332.
[2] Schneider T. and Singer J. M., *in* Phase Transition Approach to High Temperature Superconductivity (Imperial College Press, London) 2000.
[3] Schneider T., *in* The Physics of Superconductors, edited by Bennemann K. and Ketterson J. B. (Springer, Berlin) 2004, p. 111.
[4] Schneider T., *Phys. Rev. B*, **75** (2007) 174517.
[5] Schneider T., *EPL*, **79** (2007) 57005.
[6] Lee P. A. and Shenoy S. R., *Phys. Rev. Lett.*, **28** (1972) 1025.
[7] Schneider T., *Journal of Superconductivity*, **17** (2004) 41.
[8] Weyeneth S., Schneider T., Zhigadlo N.D., Karpinski J., and Keller H., *arXiv:0705.3775*, (2007) .
[9] Karpinski J., *Nature (London)*, **336** (1988) 660.
[10] Karpinski J., *Suprcond. Sci. Technol.*, **12** (1999) R153.
[11] Abrikosov A., *Sov. Phys. JETP*, **5** (1957) 1174.
[12] Prange R. E., *Phys. Rev. B*, **1** (1970) 2349.
[13] Pelissetto A. and Vicari I. E., *Physics Reports*, **368** (2002) 549.
[14] Kagawa N., Ishida T., Okuda K., Adachi S., and Tajima S., *Physica*, **C357-360** (2001) 302.
[15] Zech D., Rossel C., Lesne L., Keller H., Lee S.L., and Karpinski J., *Phys. Rev. B*, **54** (1996) 12535.
[16] Katayama K., Ishida T., Adachi S., and Tajima S., *Physica C*, **388-389** (2003) 741.
[17] Plackowski T., Wang Y., Lortz R., Junod A., and Wolf Th., *J. Phys.: Condens. Matter*, **17** (2005) 6871.
[18] Roulin M., Junod A., and Walker E., *Physica C*, **296** (1998) 137.
[19] Khasanov R., Schneider T., and Keller H., *Phys. Rev. B*, **72** (2005) 014524.